P: ISSN NO.: 2394-0344 RNI No.UPBIL/2016/67980 VOL-3* ISSUE-12* (Part-2) March- 2019
E: ISSN NO.: 2455-0817 Remarking An Analisation

Corrections to CCW Predictions for Hydromagneticshock Waves

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Abstract

The study of hydromagnetic shock waves in presence of magnetic field have been studied in this paper fromCCW[1-3] method, predictions to improve their accuracy for the motion of diverging cylindrical hydro magnetic shock waves through in an ideal gas under its own gravitating and rotating gas in the presence of constant axial magnetic field .Assuming an initial density distribution $\rho_o = \rho' r^{-w}$,where ρ', is the density at the axis of symmetry and w , is constant, the analytical expression for flow variables representing both the situations via; weak and strong cases of shock have been obtained .

Keywords: Hydromagnetic Shockwave, Rotating Gas, Gravitating Gas, Magnetic Field, Sleak and Strory Shock

Introduction

Study of hydro-magnetic shock waves has proved its importance in different branches of Science and Technology. In recent past Kumar[4-6]; and Kumar and Prakash[7-8] have assuming initial density distribution $\rho_o = \rho' r^{\pm w}$have investigated the propagation of hydro magnetic cylindrical shock waves through axial magnetic field for both cases of weak and strong shock using CCW method. J.B. Singh and S.KPandey[9]have studied the propagation of magnetogasdynamics cylindrical shock waves in self gravitating and rotating gas on the presence of constant axial magnetic field for both cases of weak and strong shock using CCW method. Assumption of any arbitrary initial density distribution of the form $\rho_o = \rho' r^{\pm w}$,$\rho_o = \rho' \exp(\pm \lambda r)$impose restriction values of propagation distance r, which are permitted by the fulfillment of initial entropy distribution condition i.e. $p_o \rho_o^{-\square}$ =c*. Study by CCW method already have two limitations viz. (i)as it takes care of state just behind the shock and (ii) for any arbitrary choice of the initial density distribution,the computational of numerical estimates of flow variables only at (psfl). Yousaf[10-12] has further,established the need to including the effect of overtaking disturbances behind the flow on the motion of shock waves, in general, has helped to Rankin interest in problems investigated before to include it to make earlier results reliable ,particularly in absence of purely developed experimental basis. In this paper, the EOD on motion of diverging cylindrical hydro magnetic shock waves through an ideal gas under its own gravitation and rotation in presence of constant axial magnetic field simultaneously, for both (i) weak and (ii) strong cases of shock, assuming an initial density decrease of the unperturbed medium as $\rho_o = \rho' r^{-w}$,the modified form of analytical expressions for flow variable have been derived.

Basic Equations

The equations governing the flow of gas enclosed by the shock front are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{\mu}{2\rho}\frac{\partial H^2}{\partial r} + \frac{Gm}{r^2} - \frac{v^2}{r} = 0$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = 0$$

$$\frac{\partial P}{\partial t} + u\frac{\partial \rho}{\partial r} - a^2\left(\frac{\partial \rho}{\partial r} + u\frac{\partial \rho}{\partial r}\right) = 0 \quad (1)$$

$$\frac{\partial m}{\partial r} - 2\pi r\rho = 0$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)(vr) = 0$$

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\frac{\partial H}{\partial r} + H\frac{u}{r} = 0$$

Where r is the radial coordinate and r, v are the radial and azimuthal components of the particle velocity. P,ρ, H, μ and m denote respectively ,the pressure, the density, the axial magnetic field, the magnetic permeabilityof the gas and mass inside the cylinder of unit cross- section of length r

Boundary Conditions

The magneto-hydrodynamic shock condition can be written in terms of single parameter $\xi=\rho/\rho_o$ as

$$\rho = \rho_o\xi \ , \ H = H_o\xi \ , \ u = \frac{\xi-1}{\xi}U \qquad (2)$$

$$U^2 = \frac{2\xi}{(\gamma+1)-(\gamma-1)\xi}\left[a_o^2 + \frac{b_o^2}{2}\{(2-\gamma)\xi+\gamma\}\right] \text{and}$$

$$P = P_o + \frac{2\rho_o(\xi-1)}{(\gamma+1)-(\gamma-1)\xi}\left[a_o^2 + \frac{\gamma-1}{4}b_o^2(\xi-1)^2\right]$$

Where 0 stands for the states immediately ahead of the shock,U is the shock velocity. a_o is the sound speed$\sqrt{\gamma P_o/\rho_o}$ and b_o is the alfven speed$\sqrt{\mu H_o^2/\rho_o}$.

Strong Shock

For the strong shock, ρ/ρ_o is large. Now consider the two cases of weak and strong magnetic field.

Case I

For weak magnetic field $b_o^2 \ll a_o^2$,using this condition the boundary conditions (2) become:

$$\rho = \rho_o\xi \ , \quad H = H_o\xi \ , \quad u = \frac{\xi-1}{\xi}U$$

$$\frac{P}{P_o} = 1 + \{\chi' a_o^2 + A' b_o^2\}\frac{U^2}{a_o^4} \ , \qquad (3)$$

Where,

$\chi' = \gamma(\xi-1)/\xi$, $A' = \frac{\gamma(\xi-1)}{4\xi}[(\gamma-1)(\xi-1)^2 - 22-\gamma\xi+\gamma$

Case II

Forstrong magnetic field at$b_o^2 \gg a_o^2$ using this condition, the boundary condition (2) becomes

$$\rho = \rho_o\xi \ , \quad H = H_o\xi \ , \quad u = \frac{\xi-1}{\xi}U$$

$$\frac{P}{P_o} = 1 + \chi\{b_o^2 + A a_o^2\}\frac{U^2}{a_o^2 b_o^2} \qquad (4)$$

Where, $\chi = \frac{\gamma(\gamma-1)(\xi-1)^3}{2\xi(2-\gamma)(\xi+\gamma)}$, $A = \frac{4}{(\gamma-1)(\xi-1)^2} - \frac{2}{(2-\gamma)\xi+\gamma}$

Characteristic Equations

The characteristic form of the system of equation (1) is easily obtained by forming a linear combination of first and third equation of system of equation (1) an only one direction in (r, t) plane can be written as

$$dP + \mu H dH + \rho c du + \frac{\rho c^2 u dr}{(u+c)r} + \frac{\rho c G m}{(u+c)}\frac{dr}{r^2} - \frac{\rho c v^2}{(u+c)}\frac{dr}{r} = 0 \qquad (5)$$

In order to estimate the strength of overtaking disturbance an independent characteristic is considered .The differential relation valid across C_+ disturbance is written by replacing c by –c in equation (5) we get

$$dP + \mu H dH - \rho c du + \frac{\rho c^2 u dr}{(u-c)r} + \frac{\rho c G m}{(u-c)}\frac{dr}{r^2} + \frac{\rho c v^2}{(u-c)}\frac{dr}{r} = 0 \qquad (6)$$

Where, $c^2 = a^2 + b^2 = \frac{\gamma P}{\rho} + \frac{\mu H^2}{\rho}$.

Analytical Relation for Flow Variables

The equilibrium state of the gas is assumed to be specified by the condition $\partial/\partial t$ =0 andH_{zo} = constant.

Using (7) the first equation of the system of the equation (1) the equilibrium prevailing condition in front of the shock can be written as-

$$\frac{1}{\rho_o}\frac{dP_o}{dr} + \frac{Gm}{r^2} - \frac{V^2}{r} = 0 \qquad (8)$$

Assuming an initial density distribution law as $\rho_o = \rho' r^{-w}$, where ρ' is the density at the axis of symmetry and w is constant.

Using $\rho_o = \rho' r^{-w}$, from the forth equation of the system of equation (1) we get

$$m = 2\pi\rho' r^{2-w}/(2-w) \qquad (9)$$

From equation (7) and (8) we get

$$\frac{P_o}{P_1} = K + K_1 r^{2-w} + K_2 r^{1-2w} \qquad (10)$$

$$\frac{da_o}{a_o} = \frac{1}{2}\left(\frac{dP_o}{P_o} + w\frac{dr}{r}\right) \qquad (11)$$

Where K′ is constant of integration $\frac{K'}{P^1} = K$, $K_1 = \frac{\Omega_o^2\rho'}{(2-w)P'}$, $K_2 = \frac{2\pi\gamma}{D(1-2w)(w-2)}$, $v = \Omega_o r$ and $D = a'^2/G\rho' P'$ is the pressure at the axis and a' is the sound velocity at the axis .

Strong Shock with Weak Magnetic Field (SSWMF)

By substituting the shock conditions (3) in to equation (5) and respective values of various quantities, we get

$$\frac{dU^2}{dr} + U^2\left[\frac{M_1}{r} + M_2\beta^2 r^{1-w} + M_3\beta^2 r^{-2w} + M_4\beta^2 r^{3-2w} + M_5\beta^2 r^{2-3w} + M_6\beta^2 r^{1-4w}\right] + M_7 r^{-w}$$
$$+ M_8 r + M_9\beta^2 r^{1-3w} - M_{10} r^{3-3w} + M_{11}\beta^2 r^{2-2w} = 0 \qquad (12)$$

Where

$M = \left[\frac{\chi'}{\gamma} + \frac{\xi'}{2}\sqrt{\frac{\chi'}{\xi}}\right]$, $N = \left[\frac{(\xi-1)}{(\xi-1)+\sqrt{\chi'\xi}} - \frac{w}{\gamma}\right]\frac{\chi'}{m}$,

$N_1 = \left[\frac{w}{\gamma} + \frac{N}{M\gamma}\right]\frac{A'\beta^2}{MK}$, $M_1 = N - N_1$

$M_2 = \left[\frac{N}{M} - 2(1-w)\right]\frac{A'K_1}{M\gamma K^2}$,

$M_3 = \frac{A'K_2}{M\gamma K^2}\left[(3w-1) + \frac{N}{M}\right]$,

$M_4 = 2A'K_1^2(2-w)/M\gamma K^3$,

$M_5 = 6K_1K_2(1-w)/M\gamma K^3$,

$M_6 = 2K_2^2(1-2w)A'/M\gamma K^3$,

$M_7 = N_2 - N_3$,

$M_8 = N_5 - N_4$,

$M_9 = N_2K_2A'/M\gamma K^2$,

$M_{10} = N_4A'/M\gamma K^2$,

$M_{11} = A'(N_2K_1 - N_4K_2)/M\gamma K^2$

$N_2 = \frac{2\pi a^2}{D(2-w)M}\left[\frac{\xi\sqrt{\chi'\xi}}{\{(\xi-1)+\sqrt{\chi'\xi}\}} - 1\right]$,

$N_3 = \beta^2A'N_2/M\gamma K$,

$N_4 = \frac{\Omega^2}{M}\left[\frac{\xi\sqrt{\chi'\xi}}{\{(\xi-1)+\sqrt{\chi'\xi}\}} - 1\right]$,

$N_5 = \beta^2A'N_4/M\gamma K$,

On integration equation (27) we get

$$\frac{U^2}{F_P} = [K_p r^{-M1} - M_{12} r^{1-w} - M_{13} r^{-2} - M_{14}\beta^2 r^{2-3w} + M_{15}\beta^2 r^{4-3w} - M_{16}\beta^2 r^{3-2w} - M_{17}\beta^2 r^{5-3w} - M_{18}\beta^2 r^{4-4w} - M_{19}\beta^2 r^{3-5w} - M_{20} r^{4-w} + M_{21}\beta^2 r^{6} - 2w = 0 \qquad (13)$$

Exp.{ $\beta^2(M_{22} r^{2-w} + M_{23} r^{1-2w} + M_{24} r^{4-2w} + M_{25} r^{3-3w} + M_{26} r^{2-4w})$} where K_p is constant of integration and M_{12}……..M_{26} are other constants.

For the C_- disturbance generated by the shock, the fluid velocity increment using may be expressed as

$du_- = \frac{\xi-1}{2\xi}[U(\frac{M_1}{r} + M_2\beta^2 r^{1-w} + M_3\beta^2 r^{-2w} + M_4\beta^2 r^{3-2w} + M_5\beta^2 r^{2-3w} + M_6\beta^2 r^{1-4w})] + \frac{1}{U}(M_7 r^{-w} + M_8 r + M_9\beta^2 r^{1-3w} - M_{10} r^{3-3w} + M_{11}\beta^2 r^{2-2w})]dr$ (14)

Substituting the shock condition (3) in to equation (6) and using the shock condition (3), we get

$du_+ = -\frac{\xi-1}{2\xi}[U(\frac{H_1}{r} + H_2\beta^2 r^{1-w} + H_3\beta^2 r^{-2w} + H_4\beta^2 r^{3-2w} + H_5\beta^2 r^{2-3w} + H_6\beta^2 r^{1-4w})] + \frac{1}{U}(H_7 r^{-w} + H_8 r + H_9\beta^2 r^{1-3w} - H_{10} r^{3-3w} + H_{11}\beta^2 r^{2-2w})]dr$ (15)

Now in presence of both C_+ and C_- disturbances the fluid velocity increment behind the shock will be related as

$$du_- + du_+ = \frac{\xi-1}{\xi} dU \quad (16)$$

using equation (14),(15) and relation(16) we get

$\frac{dU^2}{dr} + U^2[\frac{S1}{r} + S1\beta^2 r^{1-w} + S_3\beta^2 r^{-2w} + S_4\beta^2 r^{3-2w} + S_5\beta^2 r^{2-3w} + S_6\beta^2 r^{1-4w}] + S_7 r^{-w} + S_8 r + S_9\beta^2 r^{1-3w} - S_{10} r^{3-3w} + S_{11}\beta^2 r^{2-2w} = 0$ (17)

Where $S_1 = H_1+M_1$, $S_2=M_2+H_2$, $S_3=M_3+H_3$, $S_4=M_4+H_4$, $S_5=M_5+H_5$, $S_6=M_6+H_6$, $S_7=M_7+H_7$, $S_8=M_8+H_8$, $S_9=M_9+H_9$, $S_{10}=M_{10}+H_{10}$, $S_{11}=M_{11}+H_{11}$

on integration , we get

$U^2_{EOD} = [K_p^* r^{-S1} - S_{12} r^{1-w} - S_{13} r^2 - S_{14}\beta^2 r^{2-3w} + S_{15}\beta^2 r^{4-3w} - S_{16}\beta^2 r^{3-2w} - S_{17}\beta^2 r^{5-3w} - S_{18}\beta^2 r^{4-4w} - S_{19}\beta^2 r^{3-5w} - S_{20} r^{4-w} + S_{21}\beta^2 r^6 - 2w]\exp.\{\beta^2(S_{22} r^{2-w} + S_{23} r^{1-2w} + S_{24} r^{4-2w} + S_{25} r^{3-3w} + S_{26} r^{2-4w})\}$

where K_p^* is constant of integration and S_{12}……..S_{26} are other constants. (18)

Strong Shock with Strong Magnetic Field (SSSMF)

By substituting the shock conditions (4) in to equation (5) and respective values of various quantities, we get

$\frac{dU^2}{dr} + U^2[\frac{B_1}{r} + B_2\beta^{-2} r^{-2w} + B_3\beta^{-2} r^{1-w}] + B_4 r^{-w} + B_5 r - B_6\beta^{-2} r^{1-3w} + B_7\beta^{-2} r^{3-w} + B_8\beta^{-2} r^{2-2w} = 0$ (19)

Where $B = \left[\frac{\chi}{\gamma} + \frac{\xi-1}{2}\sqrt{\frac{\chi}{\xi}}\right]$, $Z = \left[\frac{\chi(\xi-1)}{B\{(\xi-1)+\sqrt{\chi\xi}\}} - \frac{\chi w}{B\gamma}\right]$, $Z_1 = \left[\frac{\chi w}{\gamma B} - Z\right]\frac{\chi A\beta^2 K}{B\gamma}$, $B_1 = Z + Z_1$

$B_2 = \frac{A\chi K_2}{B\gamma}\left[(1-3w) + \frac{wX}{\gamma B} - \frac{\chi(\xi-1)}{B\{(\xi-1)+\sqrt{\chi\xi}\}} + \frac{\chi w}{B\gamma}\right]$, $Z_5 = \frac{Z_4\,\chi A\beta^2 K}{B\gamma}$

$B_3 = \frac{A\chi K_1}{B\gamma}\left[(2-2w) + \frac{wX}{\gamma B} - \frac{\chi(\xi-1)}{B\{(\xi-1)+\sqrt{\chi\xi}\}} + \frac{\chi w}{B\gamma}\right]$, $Z_3 = \frac{Z_2\,\chi A\beta^2 K}{B\gamma}$

$B_4 = Z_2 - Z_3$, $B_5 = Z_5 - Z_4$, $B_6 = \frac{Z_2\,\chi AK_2}{B\gamma}$, $B_7 = \frac{Z_4\,\chi AK_1}{B\gamma}$, $B_8 = \frac{\chi A[Z_4 K_2 - Z_2 K_1]}{B\gamma}$

$Z_2 = \frac{2\pi a^2}{D(2-w)B}\left[\frac{\xi\sqrt{\chi\xi}}{\{(\xi-1)+\sqrt{\chi\xi}\}} - 1\right]$, $Z_4 = \frac{\Omega^2}{B}\left[\frac{\xi\sqrt{\chi\xi}}{\{(\xi-1)+\sqrt{\chi\xi}\}} - 1\right]$

On integration equation(19) ,we get

$\frac{U^2}{FP} = [K_s r^{-B1} - B_9 r^{1-w} - B_{10} r^2 - B_{11}\beta^{-2} r^{2-3w} - B_{12}\beta^{-2} r^{4-2w} - B_{13}\beta^{-2} r^{3-2w}]\exp.\{\beta^2(B_{14} r^{1-2w} + B_{15} r^{2-w})\}$ (20)

Where $K_s$is constant of integration and B_9……..B_{15} are other constants.

For the C_- disturbances generated by the shock, the fluid velocity increment using may be expressed as

$du_- = -\frac{(\xi-1)}{2\xi}[(\frac{B_1}{r} + B_2\beta^{-2} r^{-2w} + B_3\beta^{-2} r^{1-w})U + \frac{1}{U} + (B_4 r^{-w} + B_5 r - B_6\beta^{-2} r^{1-3w} + B_7\beta^{-2} r^{3-w} + B_8\beta^{-2} r^{2-2w})]dr$

(21)

Substituted the shock condition(3) in to equation(6) and again using shock condition(3), we get

$du_+ = \frac{(\xi-1)}{2\xi}[(\frac{B_1}{r} + B_2\beta^{-2} r^{-2w} + B_3\beta^{-2} r^{1-w})U + \frac{1}{U} + (B_4 r^{-w} + B_5 r - B_6\beta^{-2} r^{1-3w} + B_7\beta^{-2} r^{3-w} + B_8\beta^{-2} r^{2-2w})]dr$ (22)

Now in presence of both C_+ and C_- disturbances the fluid velocity increment behind the shock will be related as

$$du_- + du_+ = \frac{\xi-1}{\xi} dU \quad (23)$$

using equation(21),(22) and (23) we get

$\frac{dU^2}{dr} + [\frac{h_1}{r} + h_2\beta^{-2} r^{-2w} + h_3\beta^{-2} r^{1-w}]U^2 + h_4 r^{-w} + h_5 r - h_6\beta^{-2} r^{1-3w} + h_7\beta^{-2} r^{3-w} + h_8\beta^{-2} r^{2-2w} = 0$ (24)

Where $h_1 = B_1+L_1$, $h_2=B_2+L_2$, $h_3=B_3+L_3$, $h_4=B_4+L_4$, $h_5=B_5+L_5$, $h_6=B_6+L_6$, $h_7=B_7+L_7$, $h_8=B_8+L_8$

On integration (24) we get

$U^2_{EOD} = [K_s^* r^{-h1} - h_9 r^{1-w} - h_{10} r^2 - h_{11}\beta^{-2} r^{2-3w} + h_{12}\beta^{-2} r^{4-2w} - h_{13}\beta^{-2} r^{3-2w}]\exp.\{\beta^{-2}(h_{14} r^{1-2w} + h_{15} r^{2-w})\}$ (25)

Where K_s^* is constant of integration and

$h_9=h_4/1+h_1-w$, $h_{10}=h_5/(2+h_1)$, $h_{11}=[h_2h_4/(1-2w)-h_6]/2+h_1-3w$, $h_{12}=[h_3h_5/(2-w)+h_7]/4+h_1-w$

$h_{13}=[h_2h_5/(1-2w)+h_3h_4/(2-w)+h_8]/3+h_1-2w$,

$h_{14}=h_2/(2w-1)$, $h_{15}=h_3/(w-2)$

The flow variable expressions F.P. and EOD can be written by substituting (13,20) and (18,25) respectively in (3) and (4) and use for computation.

Permissible Shock Front Location (pslf)

The pressure and the density in unperturbed state given by expression (13) and $\rho_o = \rho' r^{-w}$ must fulfill the initial entropy distribution condition i.e. $P_0\rho_o^{\gamma} - c^*$ where C^* is constant, substituting the respective expression for P_0 and ρ_o,we get

$Kr^{w\gamma} + K_1 r^{2-w(1-\gamma)} + K_2 r^{1-w(1-\gamma)} - \frac{c*\rho'^{\gamma}}{p'} = 0$

This is an equation in different powers depending upon the value of w and γ of propagation distance r it may have number of roots equal to highest powers of r, in general. The roots have been evaluated by expression (26). Hit and trial method.

Result and Discussion

Numerical estimates of flow variables have been computed only at those locations of the shock front which are permitted by the initial entropy distribution condition in the unperturbed state. The results have also shown the comparison with F.P. predictions (a)(i) taking U/a_0=4.98543,5.38539,5.583945, 5.78935,6.01525 at r= 0.45, β^2=0.1, 0.15 Ω^2=10, 15 w=1.0, 1.05 γ=1.4, D=0.2, 0.3, 0.4, ξ=1.5 (ii) taking U/a_0=10.5436 at r=0.45, β^2=1.5 , γ=1.4, w=1.0 ,D=0,2 , ξ=3 (iii)taking U/a_0=15.0568 at r=0.55, β^2=1.5, γ=1.4, w=1.0 , D=0,2 ξ=4.5 (iv)taking U/a_0=20.3856 at r=0.55, β^2=1.5,

γ=1.4, w=1.0 , D=0,2 ξ=5.9 SSWMF (b)(i)taking U/a_0=7.80538, 9.5869, 9.58685, 9.78345, 9.98523 at r=0.55 β^2=10,15 Ω^2=10,15 w=1.0,1.05 , γ=1.4 , D=0.2,0.3,0.4, ξ=1.5 (ii) taking U/a_0=17.54369 at r=0.55, β^2=15, γ=1.4, w=1.0 D=0,2, ξ=3 (iii) taking U/a_0=35.89546 at r=0.55, β^2=15, γ=1.4, w=1.0 D=0,2, ξ=4.5 (iv)taking U/a_0=140.58395 at r=0.55, β^2=15, γ=1.4, w=1.0 , D=0,2 ξ=5.9 SSSWF, numerical estimates of flow variableshave been computed at permissible propagation distance for both F.P. and having included the EOD. It is observed from comparison those numerical values of flow variables, representing free propagation with the present values of flow variables included the EOD that the EOD over all retains the qualitative variation with parameter r, β^2, Ω^2, ξ,w and D unchanged.

Endnotes

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